Motivation

- Accurate fabrication (fab) performance evaluation methods are important to assess the implications of changes to design decisions, operating policies, capacity expansion efforts and market shifts.
- Flow lines have been studied for many years as models of manufacturing systems.
- More accurate, expressive, practical and computationally tractable equipment models for fab-level simulation should be developed.

System description: A flow line with redundant modules

- Wafers must receive service from each process 1, ..., M in sequence.
- P_i modules devoted to process i (labeled m_i).
- Wafer advance: Service complete & module for next process available.
- Process times are deterministic: \tau_i in module m_i.
- One module can hold at most one wafer.
- Buffers can be modeled as a process module with zero process time.

Question: When do wafers exit? How can we reduce the computation?

Max-plus algebra

“An algebra over the real numbers with maximum and addition as the two binary operations” (wikipedia)

Operations

\[ A \oplus B = \max(A, B) \]
\[ A \otimes B = A + B \]

Max-plus algebra has some similar properties to the usual algebra:

For example,

\[ (A \oplus B) \oplus C = A \oplus (B \oplus C) \]
\[ (A \oplus B) \otimes C = A \otimes (B \oplus C) \]
\[ A \oplus B = B \oplus A \]
\[ (A \oplus B) \otimes C = A \otimes C \oplus B \otimes C \]

Accomplishments

For flow lines with redundant modules

- Obtain an upper bound on the completion times for each wafer with less computation. Conjecture: The inequality is in fact equality.
- Develop a model for cluster tools which allow setups, parallel processes and lot dependent process times.

Application: Semiconductor wafer fabrication

Model for cluster tools

- Allow setups, parallel processes and lot dependent process times.
- Based on our bound/conjecture.

- Let \( E(l_i, w) \) = exit time of wafer w of lot i from the tool
- Let \( a_i \) = arrival time of lot i
- Let \( \tau_h^{(i)} \) = process time for process h of lot i
- Let \( s(i, i-1) \) = setup duration between lot i and lot i-1

Based on our bound/conjecture:

\[
E(l_i, j) = \left[ a_i \oplus \left( E(l_{i-1}, w_{i-1}) \oplus \tau_{(i-1)} \right) \right] \oplus \left( \bigoplus_{k=1}^{M} E(l_{i-k}, j) \oplus \tau_{(i-k)}^{(i)} \right) \]

with

\[ E(l_{(i,j)}) = E(l_{j,i}, W_{j,i} + j) \text{ for } j \leq 0, \]
\[ \tau_{(i-1)} = -\infty \text{ if } c(i) = c(i-1). \]

Simulation results

- Monte Carlo simulation vs. conjecture

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* Due to the random setup and arrival times

Future Directions

- For flow lines with redundant modules
- Prove the conjecture on the completion time.
- Develop tool models including additional features.